A PARTICLE METHOD WITH GLOBAL SUPERCONVERGENCE

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A method to gain a global superconvergence has been developed for meshfree methods [1, 2] and for the MPFEM – a finite element-based particle method [3]. We first discuss the definition of "superconvergence". In general a particle method approximates solution via the relation:

$$u^h(x) = \sum_{i=1}^{N} F_i(x, y_i) \cdot u_i \qquad \text{and} \qquad F_i(x, y_i) = \phi(y_i - x) \cdot c(x) \cdot \Delta v_i \tag{1}$$

where $\phi(z)$, $c(x)$, and Δv_i are window function, correction function, and a weight at y_i. The relation (1) can be interpreted as the transformation from a set of samples u_i to the approximated solution u^h through the filter *F*. The convergence rate of u^h, e.g. L_2 norm, is determined by the functional properties of $c(x)$ and $\phi(y_i - x)$ through the satisfaction of the imposed consistence conditions [4-6]. We assume that $c(x)$ is a m_c order polynomial and the degree of freedom of $\phi(z)$, i.e., the number of the undetermined constants in $\phi(z)$, is m_ϕ. Then, in general the convergence rates of (1) for $u^h(x)$ and its n^{th} derivatives, denoted as "*m*" and "m_n", respectively, are:

$$m = m_c + m_\phi + 1; \qquad m_n = m_c + m_\phi + 1 - n$$

For example, when $\phi(z)$ is defined by a 5-order spline function (m_ϕ=0) and $c(x)$ is a linear function(m_c=1), usually the L_2 norm for the meshfree methods with (1) is *2*.

Analogue to the superconvergence analysis in finite element [6], we define a particle method with superconvergence when

$$m > m_c + m_\phi + 1 \qquad \text{and/or} \qquad m_n > m_c + m_\phi + 1 - n$$

In the proposed method, the global superconvergence is achieved by a two-level interpolation scheme that allows the satisfaction of higher order reproducing condition for given m_ϕ and m_c.

References

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